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On the Stability of Communication Systems with Timed Token Protocols

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Abstract

Medium Access Control protocols for high speed LAN/MANs often employ a timer-controlled token passing mechanism to control station access to the shared media. These protocols support time-critical (real-time) applications, and provide priority among non time-critical applications. This paper considers the stability of systems with several target token rotation times. It shows that there do not exist simple, general stability bounds for these systems, because these bounds may depend on the arrival processes and the transmission time distributions not only through their means.

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1 Introduction

Communication systems which support both time-critical (real-time) applications and non time-critical applications often use a timer-controlled token passing mechanism as a Medium Access Control (MAC) protocol. During each cycle of the token every station is allowed to transmit a fixed number of time-critical frames. How many non time-critical frames a station is allowed to transmit depends on the value of the token rotation timer (TRT) of this station, which measures the time that has elapsed since the previous instant at which the token arrived at this station, and on the so-called target token rotation time (TTRT) of this station. If the token rotation timer exceeds this target then the station is not allowed to transmit any non time-critical frames at all; otherwise, the station is allowed to transmit non time-critical frames until the target token rotation time (TTRT) has been reached. The aim of this mechanism is to guarantee a certain service level to time-critical applications and to utilize the communication medium for non time-critical applications when there is little time-critical traffic. Further, it is possible to provide different priorities to various non time-critical applications by assigning different target token rotation times to the stations or to distinguishable traffic streams at the stations. Examples of MAC protocols with a timer-controlled token passing mechanism are IEEE 802.4 Token Bus, cf. [1], and FDDI (Fibre Distributed Data Interface), cf. [2, 3]; see Conti et al. [8] for a review on these protocols.

The present paper is concerned with stability conditions for systems with a timer-controlled token passing mechanism. The stability condition for systems with only time-critical traffic is well-known, and can be described by a simple expression. However, stability of systems with both types of traffic is much more difficult to establish, especially in cases with different TTRTs among the stations. It will be shown that this stability may depend on rather unusual features such as the arrival patterns of the frames.

The paper is organized as follows. Section 2 contains a more detailed description of the communication systems and the timer-controlled access protocol. For later reference, a review of stability results for systems with only time-critical traffic and for systems with only non time-critical traffic and a common TTRT for all stations is provided in Section 3. Section 4 deals with systems consisting of two stations with only non time-critical traffic and different TTRTs. Section 5 is devoted to systems consisting of two stations, one with time-critical traffic and one with non time-critical traffic. Section 6 is concerned with systems with an arbitrary number of stations with both types of traffic. The main conclusions are summarized in Section 7.

2 Description of the system and the protocol

The communication system consists of S stations, which are connected to a single channel, and a single token which is passed from station to station in cyclic order. Frames arrive at station j according to a general stationary point process with average rate λ_j, $j = 1, \ldots, S$. The total average arrival rate will be denoted by $\Lambda \doteq \sum_{j=1}^{S} \lambda_j$. In particular, we will consider batch arrival processes to reflect the bursty nature of the actual frame arrivals. Each station may contain an unbounded number of frames waiting for transmission. Frames arriving at station j have a fixed length; the transmission time of a frame at station j is constant, and will be denoted by β_j, $j = 1, \ldots, S$. The offered load at station j is defined as $\rho_j \doteq \lambda_j \beta_j$, $j = 1, \ldots, S$, and $\rho \doteq \sum_{j=1}^{S} \rho_j$ will denote the total load offered to the system. The time needed to pass the token from station $j-1$ (station 0 indicating station S) to station j is constant, and will be denoted by δ_j, $j = 1, \ldots, S$. The total ring latency is denoted by $\Delta \doteq \sum_{j=1}^{S} \delta_j$. In a few instances we will consider random (Erlang distributed) transmission and token passing times as an approximation to these constant times; in those cases, β_j and δ_j denote the respective means of these quantities, $j = 1, \ldots, S$.

For ease of the discussion it will be assumed that each station generates either time-critical or non time-critical frames of a single priority level. The set of stations with time-critical frames will be denoted by $\mathcal{T}$, that with non time-critical frames by $\mathcal{N}$. A station with several types of traffic is represented by two or more (sub)stations, one for each type of traffic. A station with time-critical traffic is allowed to transmit frames for a fixed amount of time, say τ_j for station j, $j \in \mathcal{T}$, each cycle of the token. Because frame transmission times are constant this means that station j is allowed to transmit a fixed number K_j of frames each cycle of the token, where K_j is the smallest integer such that $K_j \beta_j \geq \tau_j$, $j \in \mathcal{T}$. In the sequel, we will use the limit K_j on the number of frames rather than the limit τ_j on the total transmission time for station j, $j \in \mathcal{T}$. The use of this so-called K-limited discipline has the additional advantage that we do not have to deal with the so-called asynchronous overrun, that is, the possibly positive amount of time $K_j \beta_j - \tau_j$ which would be needed to finish transmission of a frame. For a station j with non time-critical traffic we will set $K_j \doteq \infty$, $j \in \mathcal{N}$. A station with non time-critical traffic is allowed to transmit frames during a cycle of the token only if the time that has elapsed since the previous token arrival instant at this station does not exceed the target token rotation time of this station. Because the target token rotation time should be larger than the total ring latency Δ at all stations for a proper functioning of the system, the target token rotation time at station j will be denoted by $\Delta + R_j$, $j \in \mathcal{N}$. For a station j with time-critical traffic we will set $R_j \doteq \infty$, $j \in \mathcal{T}$. If the realization of the cycle time since the previous token arrival instant is less than $\Delta + R_j$ when the token arrives again at station j then this station is allowed to start transmission of frames until the length of the foregoing cycle time plus the length of the current visit time reaches the value $\Delta + R_j$, $j \in \mathcal{N}$. In the most favorable situation, i.e., when no other station has frames to transmit, the number of frames that station j can transmit during one cycle of the token is bounded by $\hat{K}_j \doteq \lceil R_j/\beta_j \rceil$ ($\lceil x \rceil$ denotes the smallest integer larger than or equal to x); however, the number of frames that station j can transmit in two consecutive cycles is bounded by the same amount $\hat{K}_j$, $j \in \mathcal{N}$. In the case that the transmission times are the same for all stations the upper bounds R_j, $j \in \mathcal{N}$, can be chosen as multiples of the constant transmission time of a frame, so that asynchronous overrun (the amount $\hat{K}_j\beta_j - R_j$) is also avoided for stations with non time-critical traffic.

3 Review of stability results

This section contains a review of general results concerning the mean cycle time and the mean visit times for stable communication systems with a single medium. For later reference, it also discusses the stability bounds for some special cases.

A very general result for stable systems with a single communication medium and nonnegligible ring latency concerns the mean cycle time of the token. Let C denote a typical cycle time, and let V_j denote a typical visit time of the token to station j, $j = 1, \ldots, S$. Then it is clear that

$$E\{C\} = \Delta + \sum_{j=1}^{S} E\{V_j\}. \tag{1}$$

The average number of arrivals at station j during a cycle is $\lambda_j E\{C\}$; in order that the traffic at station j is stable this number of arrivals should be in balance with the average number of transmissions from station j during a cycle, which is equal to $E\{V_j\}/\beta_j$, $j = 1, \ldots, S$. Hence, the mean visit times $E\{V_j\}$, $j = 1, \ldots, S$, can be eliminated from relation (1) on condition that the whole system is stable:

$$E\{C\} = \Delta + \sum_{j=1}^{S} \lambda_j E\{C\}\beta_j = \Delta + E\{C\} \sum_{j=1}^{S} \rho_j = \Delta + E\{C\}\rho. \tag{2}$$

This implies that the mean cycle time of a stable system is given by

$$E\{C\} = \frac{\Delta}{1-\rho}. \tag{3}$$

and that $\rho < 1$ is a necessary condition for stability of the system.

First, we will discuss the stability conditions for systems with only limits on the number of frames that a station is allowed to transmit during a visit of the token, the so-called K-limited discipline. In this case, $R_j = \infty$, $j = 1, \ldots, S$. The mean visit time of the token at station j is bounded by $K_j\beta_j$, $j = 1, \ldots, S$. Hence, in case of stability of station j it should hold that

$$\rho_j E\{C\} = E\{V_j\} < K_j\beta_j, \quad j = 1, \ldots, S. \tag{4}$$

If the whole system is stable then relation (3) can be substituted into condition (4). Some straightforward rearrangements lead to the following necessary condition for stability of systems with K-limited service disciplines:

$$\rho + \Delta \max_{j=1,\dots,S} \{\lambda_j / K_j\} < 1. \tag{5}$$

The foregoing derivation is due to Kühn [11]. This condition seems to be the stability condition for general arrival processes, transmission time distributions and token-passing time distributions, with the understanding that completely deterministic systems may still be stable if the lefthand side of (5) equals 1. Condition (5) has been proved rigorously to be the necessary and sufficient stability condition for the case of Poisson arrival processes by Fricker & Jaïbi [10]. Dai & Meyn [9] prove the sufficiency of this condition for stability in a more general context via a fluid approximation of the system.

Tangemann [13] applies the foregoing reasoning of Kühn [11] to systems with token rotation timers. It states [13, Section 3] that for a station j with non time-critical traffic the following condition should hold in order that this station is stable:

$$E\{C\} + E\{V_j\} < R_j + \Delta, \quad j = 1, \dots, S. \tag{6}$$

This condition says that, on the average, the duration of a full cycle of the token plus the duration of a visit to station j should be bounded by the target token rotation time $R_j + \Delta$ for this station, $j = 1, \dots, S$. If the whole system is stable, condition (6) can be rewritten as, cf. (1)–(3),

$$(1 + \rho_j) \frac{\Delta}{1 - \rho} < R_j + \Delta, \quad j = 1, \dots, S, \tag{7}$$

or, equivalently,

$$\rho + (\rho + \rho_j) \frac{\Delta}{R_j} < 1, \quad j = 1, \dots, S. \tag{8}$$

The foregoing condition is, however, only a sufficient condition for stability, but not a necessary condition in every case. The reason is that the duration of a full cycle of the token plus the duration of a visit to station j is not necessarily bounded by the target token rotation time $R_j + \Delta$, for $j = 1, \dots, S$. The definition of the target token rotation times only implies that if for some station j, $j = 1, \dots, S$, it occurs that the realization of the passed cycle time, say $\tilde{C}$, exceeds the target, i.e., $\tilde{C} > R_j + \Delta$, then the token will pass station j, i.e., for this realization $\tilde{V}_j$ of the visit time at station j it holds that $\tilde{V}_j = 0$, but still we have $\tilde{C} + \tilde{V}_j > R_j + \Delta$. For the traffic at this station to be stable it is necessary that in some cycles $\tilde{V}_j > 0$, and hence $\tilde{C} + \tilde{V}_j \leq R_j + \Delta$ in those cycles, but this does not imply that condition (6) for the mean values must hold for every station. We will provide counterexamples in Section 4.

There are some special cases in which condition (6), and hence condition (8), must hold for every station, namely, for systems with only non time-critical traffic in which all transmission times are equal, i.e., $\beta_j = \beta$, $j = 1, \dots, S$, for some β, and all stations have the same target token rotation time and asynchronous overrun is avoided, i.e., $R_j = R \doteq M\beta$, $j = 1, \dots, S$, for some integer M. Let $\tilde{C}(n, j)$ denote the token cycle time preceding the nth visit of the token to station j, and let $\tilde{V}_j(n)$ denote the nth visit time to station j, $j = 1, \dots, S$, $n = 1, 2, \dots$. Then for the just mentioned systems it holds for every cycle n and every station j:

$$\tilde{C}(n, j) + \tilde{V}_j(n) \leq R + \Delta, \quad j = 1, \dots, S, \quad n = 1, 2, \dots. \tag{9}$$

This property can be demonstrated by the following indirect proof. Suppose that condition (9) is not satisfied for some station i in the nth cycle of the token, i.e.,

$$\tilde{C}(n,i) + \tilde{V}_i(n) > R + \Delta. \tag{10}$$

This inequality is equivalent to

$$\tilde{V}_i(n-1) + \cdots + \tilde{V}_S(n-1) + \tilde{V}_1(n) + \cdots + \tilde{V}_{i-1}(n) + \tilde{V}_i(n) > R. \tag{11}$$

Because asynchronous overrun is not possible in this case, the definition of the target token rotation time implies that station i is not allowed to transmit any frames in the nth cycle, i.e. $\tilde{V}_i(n) = 0$, and hence

$$\tilde{V}_i(n-1) + \cdots + \tilde{V}_S(n-1) + \tilde{V}_1(n) + \cdots + \tilde{V}_{i-1}(n) > R. \tag{12}$$

Since $\tilde{V}_{i-1}(n-1) \geq 0$, it follows that also

$$\tilde{V}_{i-1}(n-1) + \tilde{V}_i(n-1) + \cdots + \tilde{V}_S(n-1) + \tilde{V}_1(n) + \cdots + \tilde{V}_{i-1}(n) > R. \tag{13}$$

By the same argument as above this implies that $\tilde{V}_{i-1}(n) = 0$. In this way, one successively shows that all terms at the lefthand side of inequality (11) vanish. This clearly constitutes a contradiction, and the latter proves the validity of (9). As a consequence, condition (8) must hold for every station for such a system to be stable (strictly speaking, this condition has been deduced with a '$\leq$'–sign; as in condition (5) equality will only be relevant for completely deterministic systems). These conditions can be summarized as

$$\rho + \frac{\Delta}{R} \max_{j=1,\ldots,S} \{\rho + \rho_j\} < 1. \tag{14}$$

The foregoing stability condition is also mentioned in [12]. Altman & Liu [4, Theorem 4.3] give a rigorous proof that (14) is a necessary condition for stability of this class of systems. Further, they prove ([4, Theorem 4.1]) that the inequality $R + \Delta > 2\Delta/(1-\rho)$ is a sufficient condition for stability. The latter condition is equivalent to the condition $\rho < (R-\Delta)/(R+\Delta)$, and, hence, only meaningful if $R > \Delta$. Our numerical experiments indicate that condition (14) is also sufficient for stability of this class of systems.

When the transmission times are not equal for all stations, or when the target token rotation times are not equal for all stations, asynchronous overflow cannot be avoided, and condition (6) does not necessarily hold for all stations. Hence, the generalization of condition (14), cf. (8),

$$\rho + \Delta \max_{j=1,\ldots,S} \left\{ \frac{\rho + \rho_j}{R_j} \right\} < 1, \tag{15}$$

will only be a sufficient condition for stability in most cases. In the following sections we will study some properties of the region of stability for systems with more than one TTRT value or with mixed time-critical and non time-critical traffic.

4 Two stations with non time-critical traffic

In this section we will study the stability of a system consisting of two stations, both with non time-critical traffic, and with different target token rotation times for the two stations. The transmission

Table 1: Two-stations, only non time-critical traffic, $B_1 = 4$, $B_2 = 1$, $R_1 = 2$, $R_2 = 1$.

⋯	2	0	1	1	0	2	0	1	1	0	2	0	1	1	0	2	0	1	1	0	⋯
⋯	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	⋯

Table 2: Two-stations, only non time-critical traffic, $B_1 = 8$, $B_2 = 2$, $R_1 = 2$, $R_2 = 1$.

⋯	2	0	1	1	1	1	1	1	0	0	2	0	1	1	1	1	1	1	0	0	⋯
⋯	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	⋯

times are the same at both stations, i.e., $\beta_1 = \beta_2 = \beta$. The load of the system then becomes $\rho = \Lambda\beta$. To facilitate the study of the stability of these systems, we will assume that arrivals occur in batches of fixed sizes B_1 and B_2, simultaneously at both stations, with deterministic time intervals. The batch sizes are in the proportion $B_1 : B_2 = \lambda_1 : \lambda_2$. The interarrival times of the batches are $(B_1 + B_2)/\Lambda$. The stability condition is considered as an upper bound for the load ρ where Λ varies and the other parameters B_1, B_2, R_1, R_2, β and Δ are kept fixed. For the described deterministic arrival processes, the condition for stability can be obtained by determining ω, the average number of cycles of the token required for transmitting a frame. The stability condition then reads $\Lambda(\beta + \omega\Delta) \leq 1$, so that the stability bound ρ^* becomes

$$\rho \leq \rho^* = \beta/(\beta + \omega\Delta) = 1/(1 + \omega\Delta/\beta). \tag{16}$$

We will consider the stability bound ρ^* on the load ρ in this and the following sections in such a way that all parameters of the system are kept fixed, including the quotients λ_j/Λ, $j = 1, \ldots, S$, while the total arrival rate Λ, and hence the load ρ, increases.

We begin the discussion with an example. Suppose $\lambda_1 = 4\lambda_2$. Then, the batches have sizes $B_1 = 4L$ and $B_2 = L$ for some integer L. Consider first the case $R_1 = 2$ and $R_2 = 1$. Table 1 shows a pattern of the transmissions of the two stations in successive cycles of the token for the case $L = 1$. The system can deal with the $B_1 + B_2 = 5$ frames in 5 cycles of the token. It is not difficult to verify that the system becomes unstable when the batches arrive faster than once every 5 cycles, i.e., $\omega = 5/5 = 1$. This implies that the stability condition reads $\rho \leq \beta/(\beta + \Delta)$ in this case. Table 2 shows a pattern of the transmissions for the case $L = 2$. This pattern becomes for general L, $L \geq 2$: first two initial cycles as in Table 2, then $4L - 2$ cycles in which only one frame is transmitted by station 1, and finally $2(L - 1)$ cycles in which alternatingly one frame is transmitted by station 2 and no frame is transmitted at all. In this way, the $5L$ frames of one batch require $6L - 2$ cycles to be transmitted, so that $\omega = (6L - 2)/(5L)$. As a consequence, the stability condition for general L, $L \geq 2$, becomes $\rho \leq \rho^* = 5L\beta/(5L\beta + (6L - 2)\Delta)$. When the batch sizes increase, the stability region becomes smaller. In the limit as $L \to \infty$ the stability region shrinks to $\rho \leq 5\beta/(5\beta + 6\Delta)$. Note that station 1 is allowed to transmit first after the arrival of a batch in the patterns displayed in Tables 1 and 2; if station 2 is allowed to transmit first the transmission pattern may become somewhat different, but the minimal number of cycles for transmitting all frames of a batch turns out to be the same.

Next consider the cases $R_1 \geq 3$ and $R_2 = 1$. In these cases the minimally stable transmission patterns generally consist of two initial cycles in which the stations transmit in turn their maximally allowed amount of frames, then a series of cycles in which only station 1 transmits frames, and finally a series of cycles in which station 2 transmits a frame every second cycle; see Table 3 for an

Table 3: Two-stations, only non time-critical traffic, $B_1 = 20$, $B_2 = 5$, $R_1 = 6$, $R_2 = 1$.

⋯	6	0	5	1	5	1	2	0	0	0	0	0	0	0	0	6	0	5	1	5	1	2	0	0	⋯
⋯	0	1	0	0	0	0	0	1	0	1	0	1	0	1	0	0	1	0	0	0	0	0	1	0	⋯

Table 4: Minimal number of cycles required for transmitting a batch of frames.

		Batch sizes B_1, B_2									
R_1	R_2	4,1	8,2	12,3	16,4	20,5	24,6	28,7	32,8	36,9	40,10
2	1	5	10	16	22	28	34	40	46	52	58
3	1	4	7	12	17	21	26	31	35	40	45
4	1	3	6	10	14	18	22	26	30	34	38
5	1	2	5	9	13	16	19	23	27	31	34
6	1	2	5	8	11	15	18	21	25	28	31
7	1	2	5	7	11	13	17	20	23	27	29
8	1	2	5	7	10	13	16	19	22	25	28
9	1	2	4	7	9	13	15	19	21	24	27
10	1	2	4	7	9	12	15	17	21	23	26
∞	1	2	4	6	8	10	12	14	16	18	20
1	1	9	18	27	36	45	54	63	72	81	90
1	2	8	17	25	34	43	52	61	70	79	88
1	3	8	16	25	33	41	50	59	67	76	85
1	4	8	16	24	33	41	49	57	66	75	83
1	∞	8	16	24	32	40	48	56	64	72	80

example. The length of the series of cycles in which only station 1 transmits depends on divisibility properties of the batch size B_1 and the target R_1. As a consequence, the stability bound is not necessarily a monotonous decreasing function of L. For example, in the case $R_1 = 3$, $R_2 = 1$, the stability region is maximal for $L = 2$, and it is larger for $L = 3\ell + 2$ than for $L = 3\ell + 1$, $\ell = 0, 1, 2, \ldots$, but otherwise the tendency is that this region shrinks with increasing L. Table 4 shows the minimal number of cycles $\omega \times (B_1 + B_2)$ that are required for transmitting batches of various sizes for protocols with either $R_2 = 1$ or $R_1 = 1$. The required number of cycles decreases with increasing R_1 (respectively R_2) until the minimal value associated with a given batch size is reached. The latter happens when the target token rotation time of one station is so large that this station can transmit a whole batch of frames without hindrance by the other station.

Finally, consider the cases $R_1 \geq 2$, $R_2 \geq 2$. The determination of the minimal number of cycles for a given batch of frames becomes much more complicated, because this number is in general no longer an integer. Starting from an empty system there may be first a transient sequence of cycles before a recurrent series of cycles is reached. It is possible that a backlog of frames is built up during the transient sequence. The length of the transient sequence and the size of the backlog may depend on the individual token passing times and on the initial position of the token. Moreover, we have found cases in which the length of the recurrent series of cycles depends on the total ring latency. Table 5 illustrates this sensitivity of the stability bound. It concerns a system with transmission times $\beta = 1$. In this table, n_T stands for the number of cycles that pass before a recurrent series of cycles occurs, n_R is the number of cycles in such a recurrent series of cycles, n_B is the number

Table 5: Two-stations, only non time-critical traffic, $B_1 = 4$, $B_2 = 1$, $R_1 = 3$, $R_2 = 4$.

Token-passing times			Token initially at station 1						Token initially at station 2					
δ_1	δ_2	Δ	n_T	n_R	n_B	n_L	ω	ρ^*	n_T	n_R	n_B	n_L	ω	ρ^*
0.2	0.2	0.4	10	8	3	0	.5333	.8242	36	8	3	1	.5333	.8242
2.0	0.0	2.0	13	8	3	1	.5333	.4839	33	8	3	3	.5333	.4839
1.0	1.0	2.0	42	8	3	3	.5333	.4839	51	8	3	4	.5333	.4839
0.0	2.0	2.0	50	8	3	4	.5333	.4839	51	8	3	4	.5333	.4839
4.0	0.0	4.0	19	14	5	0	.5600	.3086	80	14	5	4	.5600	.3086
2.0	2.0	4.0	38	14	5	2	.5600	.3086	137	14	5	8	.5600	.3086
0.0	4.0	4.0	137	14	5	7	.5600	.3086	138	14	5	7	.5600	.3086
8.0	0.0	8.0	31	26	9	0	.5778	.1779	244	26	9	8	.5778	.1779
4.0	4.0	8.0	114	26	9	3	.5778	.1779	454	26	9	15	.5778	.1779
0.0	8.0	8.0	453	26	9	15	.5778	.1779	454	26	9	15	.5778	.1779
40.0	0.0	40.0	127	122	41	0	.5951	.0403	5012	122	41	40	.5951	.0403
20.0	20.0	40.0	2450	122	41	19	.5951	.0403	9894	122	41	79	.5951	.0403
0.0	40.0	40.0	9893	122	41	79	.5951	.0403	9894	122	41	79	.5951	.0403

Table 6: Two-stations, only non time-critical traffic, $R_1 \geq 2R_2$.

R_1	0	$R_1 - R_2$	R_2	$R_1 - R_2$	R_2	$\cdots$	X	0	0	0	0	$\cdots$
0	R_2	0	0	0	0	$\cdots$	Y	$R_2 - Y$	Y	$R_2 - Y$	Y	$\cdots$

of batch arrivals during a recurrent series of cycles, and n_L is the total number of frames that is eventually continuously present in the system. The average number of cycles required per frame becomes $\omega = n_R/(n_B \times (B_1 + B_2))$, and the stability bound follows with (16). The results displayed in this table (and also those of the Tables 9, 12, 14 and 17) have been obtained by long deterministic simulations (long with respect to the given values of $n_T + n_R$) with, ultimately, small step sizes in the (rational) values of ω. It should be noted that the sensitivity of ω with respect to the ring latency seems to be more exception than rule. For instance, in the same case as considered in Table 5 but with batch sizes $B_1 = 4L$, $B_2 = L$, $L \geq 2$, ω does not depend on Δ. In spite of this possible sensitivity of ω with respect to Δ its limiting value as $L \to \infty$ does not depend on Δ in general, and can be established unambiguously and more easily.

In the second part of this section we will derive formulas for the limiting value of the average number of cycles of the token required per frame as the batch sizes increase for general (integer) ratios between the batch sizes.

First, consider the case $R_1 \geq 2R_2$, cf. Table 6. Both stations start in a greedy way, but after two cycles station 1 is able to monopolize the token until this station has transmitted all its frames. In every two cycles station 1 can transmit R_1 frames. After station 1 has transmitted its frames station 2 can take over, and it can transmit R_2 frames every two cycles. When frames arrive simultaneously with deterministic intervals in large batches of sizes proportional as $\lambda_1 : \lambda_2$, say of sizes $\lambda_1 L$ and $\lambda_2 L$, then it takes $2L(\lambda_1/R_1 + \lambda_2/R_2) + o(L)$ cycles to transmit all $(\lambda_1 + \lambda_2)L = \Lambda L$ frames, asymptotically as $L \to \infty$. The $o(L)$ term stems from the first two "greedy" cycles, the transitional cycle in which station 1 transmits its last X frames and station 2 can transmit $Y = [R_2 - X]^+$ frames (with $[x]^+ \doteq \max\{0, x\}$), cf. Table 6, and the last cycle. This term is in fact bounded as

Table 7: Two-stations, only non time-critical traffic, $R_2 \le R_1 < 2R_2$.

R_1	0	$R_1 - R_2$	R_2	$R_1 - R_2$	$R_1 - R_2$	R_2	$R_1 - R_2$	$R_1 - R_2$	$\cdots$
0	R_2	0	0	$2R_2 - R_1$	0	0	$2R_2 - R_1$	0	$\cdots$

Table 8: Two-stations, only non time-critical traffic, $\frac{1}{2}R_2 < R_1 \le R_2$.

R_1	0	0	$2R_1 - R_2$	0	0	$2R_1 - R_2$	0	0	$\cdots$
$R_2 - R_1$	R_1	$R_2 - R_1$	$R_2 - R_1$	R_1	$R_2 - R_1$	$R_2 - R_1$	R_1	$R_2 - R_1$	$\cdots$

$L \to \infty$. In this case, the average number of cycles per frame becomes, asymptotically as $L \to \infty$,

$$\omega = \frac{2}{\Lambda}\left(\frac{\lambda_1}{R_1} + \frac{\lambda_2}{R_2}\right). \tag{17}$$

Next, consider the case $R_2 \le R_1 < 2R_2$, cf. Table 7. After two initial cycles there appears a periodic behavior. As long as both stations have sufficiently many frames to transmit, every three cycles of the token station 1 can transmit $2R_1 - R_2$ frames and station 2 can transmit $2R_2 - R_1$ frames. This implies that if $\lambda_1 : \lambda_2 = 2R_1 - R_2 : 2R_2 - R_1$, then the stations are in balance. Hence, define

$$\alpha_N \doteq \frac{2\lambda_1 + \lambda_2}{\lambda_1 + 2\lambda_2}. \tag{18}$$

If $\alpha_N R_2 < R_1 < 2R_2$ then station 1 will be first exhausted of frames, in $3\lambda_1 L/(2R_1 - R_2) + o(L)$ cycles; afterwards, station 2 will have to transmit its remaining frames with an intensity of R_2 frames every two cycles. In this case, the average number of cycles per frame becomes asymptotically

$$\omega = \frac{1}{\Lambda}\left\{\frac{3\lambda_1}{2R_1 - R_2} + \frac{2}{R_2}\left[\lambda_2 - \lambda_1\frac{2R_2 - R_1}{2R_1 - R_2}\right]\right\} = \frac{1}{\Lambda}\frac{\lambda_1 + 2\lambda_2}{R_2}. \tag{19}$$

Note that (19), which does not depend on the value of R_1, coincides with (17) when $R_1 = 2R_2$. The periodic pattern becomes as in Table 8 when $\frac{1}{2}R_2 < R_1 \le R_2$, but the amounts of frames transmitted in every three cycles remains the same as when $R_2 \le R_1 < 2R_2$. Hence, if $\frac{1}{2}R_2 < R_1 < \alpha_N R_2$ then station 2 will be first exhausted of frames, and station 1 will have to transmit its remaining frames with an intensity of R_1 frames every two cycles. In this case, the average number of cycles per frame becomes asymptotically

$$\omega = \frac{1}{\Lambda}\left\{\frac{3\lambda_2}{2R_2 - R_1} + \frac{2}{R_1}\left[\lambda_1 - \lambda_2\frac{2R_1 - R_2}{2R_2 - R_1}\right]\right\} = \frac{1}{\Lambda}\frac{2\lambda_1 + \lambda_2}{R_1}. \tag{20}$$

Note that (20), which does not depend on the value of R_2, is equivalent to (19) when $R_1 = \alpha_N R_2$. Finally, in the case $R_1 \le \frac{1}{2}R_2$ the pattern becomes the opposite of that of Table 6, i.e., station 2 is able to monopolize the token until this station has transmitted all its frames. The average time per frame is asymptotically given by (17), which coincides with (20) when $R_1 = \frac{1}{2}R_2$.

Table 9 shows the stability bound ρ^* for several batch sizes in the proportion $B_1 : B_2 = 4 : 1$ and for several values of the TTRTs, for the case $\delta_1 = \delta_2 = \frac{1}{2}\Delta$ and $\Delta/\beta = 0.40$. The third column indicates at which station the traffic becomes unstable (u.s.) when the load exceeds the given bound. In the column with the header "limit" the limit of ρ^* as $L \to \infty$ is listed for batch sizes $B_1 = 4L$, $B_2 = L$, cf. (17), (19), (20). The bound from the sufficient condition (15) is shown

Table 9: Stability bound ρ^* for a two-station system with only non time-critical traffic.

R_1	R_2	u.s.	4,1	8,2	12,3	16,4	20,5	24,6	28,7	32,8	limit	SC	PSA
1	1	1	.5814	.5814	.5814	.5814	.5814	.5814	.5814	.5814	.5814	.5814	.5814
2	1	2	.7143	.7143	.7009	.6944	.6906	.6881	.6863	.6849	.6757	.6757	.6976
3	1	2	.7576	.7813	.7576	.7463	.7485	.7426	.7384	.7407	.7282	.6757	.7509
4	1	2	.8065	.8065	.7895	.7813	.7764	.7732	.7709	.7692	.7576	.6757	.7792
5	1	2	.8621	.8333	.8065	.7937	.7962	.7979	.7919	.7874	.7764	.6757	.7964
6	1	2	.8621	.8333	.8242	.8197	.8065	.8065	.8065	.8000	.7895	.6757	.8076
7	1	2	.8621	.8333	.8427	.8197	.8278	.8152	.8140	.8130	.7991	.6757	.8155
8	1	2	.8621	.8333	.8427	.8333	.8278	.8242	.8216	.8197	.8065	.6757	.8212
9	1	2	.8621	.8621	.8427	.8475	.8278	.8333	.8216	.8264	.8123	.6757	.8254
10	1	2	.8621	.8621	.8427	.8475	.8389	.8333	.8373	.8264	.8170	.6757	.8287
1	2	1	.6098	.5952	.6000	.5952	.5924	.5906	.5892	.5882	.5814	.5814	.5991
2	2	1	.7353	.7353	.7353	.7353	.7353	.7353	.7353	.7353	.7353	.7353	.7353
3	2	1,2	.8065	.8065	.8065	.8065	.8065	.8065	.8065	.8065	.8065	.8065	.8065
4	2	2	.8333	.8333	.8333	.8333	.8224	.8242	.8178	.8197	.8065	.8065	.8241
5	2	2	.8621	.8621	.8523	.8475	.8446	.8427	.8413	.8403	.8278	.8065	.8454
6	2	2	.8772	.8621	.8721	.8772	.8562	.8621	.8578	.8547	.8427	.8065	.8598
1	3	1	.6098	.6098	.6000	.6024	.6039	.6000	.5973	.5988	.5906	.5814	.6029
2	3	1	.7576	.7463	.7426	.7463	.7396	.7426	.7384	.7389	.7353	.7353	.7459
3	3	1	.8065	.8065	.8065	.8065	.8065	.8065	.8065	.8065	.8065	.8065	.8065
4	3	1	.8475	.8475	.8475	.8475	.8475	.8475	.8475	.8475	.8475	.8475	.8475
5	3	2	.8696	.8681	.8687	.8681	.8681	.8687	.8689	.8696	.8621	.8621	.8667
6	3	2	.8824	.8824	.8824	.8824	.8803	.8824	.8750	.8734	.8621	.8621	.8765
1	4	1	.6098	.6098	.6098	.6024	.6039	.6048	.6055	.6024	.5952	.5814	.6037
2	4	1	.7576	.7576	.7500	.7463	.7485	.7500	.7447	.7463	.7353	.7353	.7504
3	4	1	.8242	.8130	.8123	.8108	.8106	.8094	.8102	.8091	.8065	.8065	.8132
4	4	1	.8475	.8475	.8475	.8475	.8475	.8475	.8475	.8475	.8475	.8475	.8475
5	4	1	.8741	.8741	.8741	.8741	.8741	.8741	.8741	.8741	.8741	.8741	.8741
6	4	1,2	.8929	.8929	.8929	.8929	.8929	.8929	.8929	.8929	.8929	.8929	.8929

in the column with the header "SC". Observe that this sufficient bound may be very conservative if $R_1 \gg R_2$. Finally, in the last column the estimated stability bound is displayed for the case of Poisson arrival processes with single arrivals. These bounds have been obtained with the aid of the power-series algorithm (PSA). This algorithm has been described in detail for the present class of communication systems in Blanc & Lenzini [7]. For reviews on the PSA see Blanc [5, 6]. For application of the PSA it is necessary to approximate the constant transmission times and token-passing times by Erlang distributed random variables. For Table 9, we have used Erlang E_4 distributions for the transmission times and Erlang E_2 distributions for the token-passing times between the stations. In numerical experiments with the PSA instability can be detected by the occurrence of negative state probabilities. This is illustrated in Tables 10 and 11. These tables show zero probabilities and means for the random variables N_j, the number of frames present at station j, $j = 1, 2$, for the case $R_1 = 2$, $R_2 = 1$. It is clear from these tables that station 2 is the first to become unstable. Table 11 concerns the model that was used for the estimations for the stability bound in Table 9, i.e., with Erlang E_4 distributed transmission times and ring latency. It yields an estimated $\rho^* \approx 0.6976$. Table 10 concerns a similar model, but with exponentially distributed transmission and token-passing times. It yields an estimated $\rho^* \approx 0.6959$. We note that the values for performance measures produced by the PSA for $\rho > \rho^*$ are extrapolations of the measures for

Table 10: Two-stations, only non time-critical traffic, $R_1 = 2$, $R_2 = 1$: exponential case.

ρ	$\Pr\{N_1 = N_2 = 0\}$	$\Pr\{N_1 = 0\}$	$\Pr\{N_2 = 0\}$	$E\{N_1 + N_2\}$	$E\{N_1\}$	$E\{N_2\}$
0.692	6.93e-3	0.1785	1.96e-2	120.261	3.205	117.057
0.693	5.18e-3	0.1772	1.47e-2	161.266	3.224	158.042
0.694	3.42e-3	0.1760	9.72e-3	244.474	3.244	241.231
0.695	1.66e-3	0.1747	4.73e-3	504.122	3.263	500.859
0.696	-1.00e-4	0.1734	-2.86e-4	-8373.037	3.283	-8376.326

Table 11: Two-stations, only non time-critical traffic, $R_1 = 2$, $R_2 = 1$: Erlang E_4 case.

ρ	$\Pr\{N_1 = N_2 = 0\}$	$\Pr\{N_1 = 0\}$	$\Pr\{N_2 = 0\}$	$E\{N_1 + N_2\}$	$E\{N_1\}$	$E\{N_2\}$
0.694	6.52e-3	0.1902	2.03e-2	100.420	2.554	97.866
0.695	4.70e-3	0.1889	1.47e-2	139.588	2.569	137.019
0.696	2.87e-3	0.1877	8.99e-3	228.693	2.584	226.109
0.697	1.04e-3	0.1864	3.27e-3	631.170	2.599	628.572
0.698	-7.90e-4	0.1851	-2.49e-3	-832.581	2.614	-835.194

stable systems based on the assumption that they are regular functions of ρ. They have no meaning for the actual system, also not those of $\Pr\{N_1 = 0\}$ and $E\{N_1\}$. The latter measures could be defined for some values $\rho > \rho^*$, since station 1 remains stable, but they are not differentiable at $\rho = \rho^*$ while the PSA computes analytical continuations beyond ρ^*. Still, these values are useful for determining ρ^*. The foregoing and other examples indicate that the stability bound ρ^* may also vary with the transmission and token-passing time distributions for this type of communication systems. Probably, the stability bound for the case of Poisson arrivals and constant transmission and token-passing times is still somewhat larger than in the case of Erlang E_4 distributions. It is rather surprising that the stability bound for the case of Poisson arrivals can be considerably larger than that for the corresponding case of deterministic, large batch arrivals. The example considered in Table 9 and other examples indicate that the stability bound ρ^* is independent of the batch sizes, and also of the choice of probability distributions, not only in the cases $R_1 = R_2$ in agreement with (14), but more generally for $R_2 \leq R_1 \leq \alpha_N R_2$ ($\alpha_N = \frac{3}{2}$ in the example of Table 9).

The examples in Table 9 might suggest that the limiting value of the stability bound ρ^* corresponding to the case of large batch sizes acts as a lower bound for the stability bound for the cases of finite batch sizes. Although this is true for many cases we examined, we have also found cases in which this property does not hold. Typically, these counterexamples concern cases with large TTRT values and/or relatively large ring latency in comparison with the transmission times. Table 12 shows such a counterexample. In this example, $\delta_1 = \delta_2 = \frac{1}{2}\Delta$ and $\Delta/\beta = 8.0$. The stability bound ρ^* reaches an upper bound with increasing values of R_1 which is smaller than the corresponding upper bound for the large batch size limit of the stability bound, for batch sizes $B_1 = 4L$, $B_2 = L$, and L odd (Table 12 shows a transition point at $R_1 = 16$ for $L = 1$ and one at $R_1 = 40$ for $L = 5$). In spite of these counterexamples, the large batch size limit of ρ^* remains a good starting point to search for the stability bound for finite batch sizes.

Finally we remark that the stability bound may change if frames arrive one by one with constant interarrival times instead of in batches. For instance, in the case $R_1 = 4$, $R_2 = 1$ and $\lambda_1 : \lambda_2 = 4 : 1$ station 1 may not accumulate sufficiently many frames to fully use its target amount R_1 but may

Table 12: Stability bound ρ^* for a two-station system with non time-critical traffic, $R_2 = 4$.

batch sizes	R_1 6	8	10	12	16	20	24	32	40	80	120	∞
4,1	.2941	.3333	.3846	.3846	.3846	.3846	.3846	.3846	.3846	.3846	.3846	.3846
8,2	.2941	.3333	.3846	.4167	.4545	.5556	.5556	.5556	.5556	.5556	.5556	.5556
12,3	.2941	.3333	.3600	.4054	.4839	.4839	.5172	.5172	.5172	.5172	.5172	.5172
16,4	.2941	.3333	.3846	.3846	.4545	.5556	.5556	.5556	.5556	.5556	.5556	.5556
20,5	.2941	.3333	.3731	.4237	.4237	.4717	.5102	.5102	.5102	.5102	.5102	.5102
40,10	.2941	.3165	.3521	.3731	.4237	.4545	.4902	.4902	.5102	.5556	.5556	.5556
limit	.2941	.2941	.3247	.3488	.3846	.4098	.4286	.4545	.4717	.5102	.5245	.5556

Table 13: Transmission pattern: batch arrivals (left) vs. individual arrivals (right); $R_1 = 4$, $R_2 = 1$.

$\cdots$	4	0	0	4	0	0	$\cdots$
$\cdots$	0	1	0	0	1	0	$\cdots$

$\cdots$	1	0	2	1	1	0	2	1	$\cdots$
$\cdots$	0	1	0	0	0	1	0	0	$\cdots$

still be able to block station 2 completely. Table 13 shows the transmission pattern for the cases of simultaneous batch arrivals with $B_1 = 4$, $B_2 = 1$ and of individual arrivals with constant intervals at station 1. In this example with frames arriving one by one and $\Delta/\beta = 0.4$ we find $\rho^* = 0.7576$. This equals the large batch size limit, cf. Table 9. This suggest that the token rotation timer mechanism performs better with bursty traffic than with more evenly spread out traffic in some circumstances. Also, a phase difference between the arrival patterns of the (two) stations might influence the stability bound. This issue will not be elaborated upon.

5 Two stations with mixed traffic

In this section we consider the stability of systems with two stations, one with time-critical and one with non time-critical traffic. Station 1 generates time-critical traffic, and is allowed to transmit K_1 frames per cycle of the token. Station 2 generates non time-critical traffic, and has a target token rotation time of $R_2 + \Delta$. The transmission times are the same at both stations, i.e., $\beta_1 = \beta_2 = \beta$. We will again consider the case that arrivals occur in batches of fixed sizes in the proportion $B_1 : B_2 = \lambda_1 : \lambda_2$, simultaneously at both stations, with fixed time intervals. As in Section 4, it is rather straightforward to determine the asymptotic value of the number of cycles required to transmit a frame, ω, as the batch sizes become large.

If $K_1 \geq R_2$ then station 2 is blocked as long as station 1 has frames to transmit. Station 1 can first transmit K_1 frames per cycle, afterwards station 2 can transmit R_2 frames every two cycles. Hence, the average number of cycles per frame becomes asymptotically

$$\omega = \frac{1}{\Lambda}\left(\frac{\lambda_1}{K_1} + 2\frac{\lambda_2}{R_2}\right). \tag{21}$$

If $K_1 < R_2$ then as long as station 1 has frames to transmit, this station can transmit K_1 frames per cycle, while station 2 can transmit $R_2 - K_1$ frames every two cycles. The stations are in balance if they need the same number of cycles to transmit a batch of frames, that is, if

$$\frac{\lambda_1 L}{K_1} = \frac{2\lambda_2 L}{R_2 - K_1}. \tag{22}$$

Table 14: Stability bound ρ^* for a two-station system with mixed traffic.

K_1	R_2	u.s.	4,1	8,2	12,3	16,4	20,5	24,6	28,7	32,8	limit	SC	PSA
1	1	2	.7143	.6944	.6881	.6849	.6831	.6818	.6809	.6802	.6757	.6757	.6953
2	1	2	.8065	.7813	.7732	.7692	.7669	.7653	.7642	.7634	.7576	.6757	.7718
3	1	2	.8065	.8065	.8065	.7937	.7962	.7979	.7919	.7937	.7895	.6757	.7997
4	1	2	.8621	.8333	.8242	.8197	.8170	.8152	.8140	.8130	.8065	.6757	.8137
5	1	2	.8621	.8333	.8242	.8197	.8278	.8242	.8216	.8197	.8170	.6757	.8220
6	1	2	.8621	.8333	.8427	.8333	.8278	.8333	.8294	.8264	.8242	.6757	.8274
1	2	1	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576
2	2	2	.8333	.8333	.8242	.8197	.8170	.8152	.8140	.8130	.8065	.8065	.8220
3	2	2	.8621	.8621	.8621	.8475	.8503	.8523	.8454	.8475	.8427	.8065	.8553
4	2	2	.8929	.8929	.8824	.8772	.8741	.8721	.8706	.8696	.8621	.8065	.8725
5	2	2	.8929	.8929	.8824	.8772	.8865	.8824	.8794	.8772	.8741	.8065	.8830
6	2	2	.8929	.8929	.9036	.8929	.8865	.8929	.8883	.8850	.8824	.8065	.8899
1	3	1	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576
2	3	1,2	.8621	.8621	.8621	.8621	.8621	.8621	.8621	.8621	.8621	.8621	.8621
3	3	2	.8824	.8772	.8824	.8696	.8681	.8721	.8663	.8658	.8621	.8621	.8750
4	3	2	.9036	.9036	.9036	.8982	.8950	.8929	.8913	.8902	.8824	.8621	.8934
5	3	2	.9091	.9091	.9036	.9009	.9080	.9036	.9005	.8969	.8950	.8621	.9047
6	3	2	.9259	.9091	.9259	.9146	.9058	.9146	.9099	.9048	.9036	.8621	.9122
1	4	1	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576	.7576
2	4	1	.8621	.8621	.8621	.8621	.8621	.8621	.8621	.8621	.8621	.8621	.8621
3	4	2	.8974	.8929	.8964	.8969	.8961	.8964	.8974	.8955	.8929	.8929	.8956
4	4	2	.9091	.9091	.9091	.9091	.9058	.9036	.9021	.9009	.8929	.8929	.9040
5	4	2	.9191	.9174	.9146	.9091	.9191	.9146	.9099	.9091	.9058	.8929	.9156
6	4	2	.9259	.9259	.9317	.9259	.9191	.9259	.9211	.9174	.9146	.8929	.9235

This condition can be reformulated as $K_1 = \alpha_M R_2$, with

$$\alpha_M \doteq \frac{\lambda_1}{\lambda_1 + 2\lambda_2}. \tag{23}$$

If $\alpha_M R_2 \leq K_1 < R_2$, then station 1 is first exhausted of frames, and station 2 can transmit its remaining frames with an intensity of R_2 frames every two cycles. The average number of cycles per frame becomes asymptotically, independently of K_1,

$$\omega = \frac{1}{\Lambda}\left\{\frac{\lambda_1}{K_1} + \frac{2}{R_2}\left[\lambda_2 - \lambda_1\frac{R_2 - K_1}{2K_1}\right]\right\} = \frac{1}{\Lambda}\frac{\lambda_1 + 2\lambda_2}{R_2}. \tag{24}$$

If $K_1 < \alpha_M R_2$, then station 2 is first exhausted of frames. The number of cycles required for transmitting a batch of frames is just the number of cycles that station 1 needs to transmit its frames, that is,

$$\omega = \frac{1}{\Lambda}\frac{\lambda_1}{K_1}. \tag{25}$$

It is readily verified that the asymptotic value of ω for large batch sizes, cf. (21), (24), (25), can be summarized for all values of K_1 and R_2 as

$$\omega = \frac{1}{\Lambda}\left\{\frac{\lambda_1}{K_1} + \frac{1}{R_2}\left[2\lambda_2 - \frac{\lambda_1}{K_1}[R_2 - K_1]^+\right]^+\right\}. \tag{26}$$

Table 15: Three stations, one with non time-critical traffic.

K_1	$\cdots$	K_1	0	0	$\cdots$	0	0	0	0	$\cdots$	0	0
K_2	$\cdots$	K_2	K_2	K_2	$\cdots$	K_2	K_2	0	0	$\cdots$	0	0
0	$\cdots$	0	$R_3 - K_2$	0	$\cdots$	$R_3 - K_2$	0	R_3	0	$\cdots$	R_3	0

Table 14 shows the stability bound ρ^* for a similar system as to which Table 9 is devoted, namely $B_1 : B_2 = 4 : 1$, $\delta_1 = \delta_2 = \frac{1}{2}\Delta$ and $\Delta/\beta = 0.40$, but with one station with time-critical and one with non time-critical traffic. The column with the header "SC" contains a sufficient bound for stability which is obtained by combining conditions (6) for station 1 and (8) for station 2, that is,

$$\rho + \Delta \max\left\{\frac{\lambda_1}{K_1}, \frac{\rho + \rho_2}{R_2}\right\} < 1. \tag{27}$$

It turns out that the stability bound ρ^* is independent of the arrival process for $K_1 \leq \alpha_M R_2$ ($\alpha_M = \frac{2}{3}$ in the example of Table 14). This feature will be due to the fact that station 1 becomes unstable first in these cases, and because the stability condition for systems with K-limited service is robust with respect to the arrival processes and the shape of probability distributions.

6 Systems with more than two stations

This section is devoted to systems with mixed traffic and an arbitrary number of stations. We restrict the discussion to the practically important case that all transmission times are equal, i.e. $\beta_j = \beta$, $j = 1, \ldots, S$.

When a communication system with timer controlled protocol consists of more than two traffic streams (stations) it becomes more complicated to determine even the asymptotic value of the required number of cycles per frame, ω, as the batch sizes become large, because many combinations of relative batch sizes and service limits and target token rotation times have to be dealt with separately. An exception is formed by systems with $S-1$ stations with time-critical traffic and only one station with non time-critical traffic. In the case of $S = 3$ stations, and when the stations with time-critical traffic are ordered such that $\lambda_1/K_1 \leq \lambda_2/K_2$, implying that station 1 does not need more cycles to transmit all its frames than station 2, then formula (26) can be generalized to

$$\omega = \frac{1}{\Lambda}\left\{\frac{\lambda_2}{K_2} + \frac{1}{R_3}\left[2\lambda_3 - \frac{\lambda_1}{K_1}[R_3 - K_1 - K_2]^+ - \left(\frac{\lambda_2}{K_2} - \frac{\lambda_1}{K_1}\right)[R_3 - K_2]^+\right]^+\right\}. \tag{28}$$

For instance, a typical transmission pattern for the case that the TTRT of station 3 is such that $K_2 < R_3 \leq K_1 + K_2$ and the arrival rate at station 3 is so large that this station is the last to be exhausted of frames (i.e., $2\lambda_3 > (R_3 - K_2)[(\lambda_1/K_1) - (\lambda_2/K_2)]$) is shown in Table 15. Expression (28) is readily generalized further to systems with $S = 4, 5, \ldots$ stations.

For general systems the asymptotic value of ω can be obtained by an iterative procedure with S stages at the most. At each stage it is determined which of the stations with frames still to be transmitted will be exhausted first. Then, the number of cycles it takes to transmit all the remaining frames of this specific station is added to the total number of cycles required in the previous stages, and is used to update the remaining number of frames at the other stations. For stations with time-critical traffic it is readily found that they are exhausted of frames after about

Table 16: Transmission pattern; one station with time-critical, five with non time-critical traffic.

4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	$\cdots$
6	0	0	1	1	2	3	1	1	1	2	3	1	1	1	2	3	1	1	1	2	3	$\cdots$
0	6	0	0	1	1	2	3	1	1	1	2	3	1	1	1	2	3	1	1	1	2	$\cdots$
0	0	5	0	0	0	0	1	2	0	0	0	1	2	0	0	0	1	2	0	0	0	$\cdots$
0	0	0	4	0	0	0	0	1	2	0	0	0	1	2	0	0	0	1	2	0	0	$\cdots$
0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\cdots$

$\lambda_j L/K_j$ cycles. In each stage, it has to be determined which stations with non time-critical traffic are able to transmit, and if so, how many frames per cycle. This can be done by the following procedure. First compute M, the sum of the service limits of the stations with time-critical traffic that are not yet exhausted. For ease of the discussion suppose that there are Z not yet exhausted stations with non time-critical traffic numbered such that $R_1 \geq R_2 \geq \cdots \geq R_Z$ (if this is not the case, a permutation has to be used; the position of the stations on the ring is not important here). Then, determine the largest index J with the property

$$(J+1)R_J > M + \sum_{i=1}^{J} R_i. \tag{29}$$

In this stage, stations $J+1, \ldots, Z$ are able to transmit at most a negligible number of frames with respect to the batch sizes, while station j is able to transmit $(J+1)R_j - M - \sum_{i=1}^{J} R_i$ frames every $J+1$ cycles, $j = 1, \ldots, J$. The foregoing statements are generalizations of those discussed in Sections 4 and 5. We shall illustrate them by a numerical example, cf. Table 16. Here, $M = 4$ and all time-critical traffic is concentrated in a single station "0" (this is no restriction for our purpose). Further, there are five stations with non time-critical traffic, with targets $R_1 = R_2 = 10$, $R_3 = R_4 = 9$, $R_5 = 8$. It is readily verified that $J = 4$, cf. (29). The transmission pattern in Table 16 shows that station 5 is only able to transmit 2 frames during the initial 5 cycles. Station 1 and 2 are able to transmit 8 frames in the recurrent sequence of 5 cycles, while stations 3 and 4 are able to transmit 3 frames in this sequence of cycles, in agreement with the above statements.

This section is concluded with some numerically obtained stability bounds for cases with finite batch sizes. Table 17 concerns a system with 12 traffic streams. Stations 1,4,7 and 10 generate time-critical traffic; they are assigned a frame limit of $K_{3i-2} = 2$, $i = 1, \ldots, 4$. The other stations generate non time-critical traffic. There are two priority levels for this type of traffic: $R_{3i-1} = 10$ and $R_{3i} = 5$, $i = 1, \ldots, 4$. This system can be viewed as consisting of four superstations, each with three types of traffic: time-critical, and high and low priority non time-critical traffic. We assume that there are only non-negligible token-passing times between these superstations, i.e., $\delta_{3i-2} = \frac{1}{4}\Delta$, $i = 1, \ldots, 4$, and not between the traffic streams within the superstations. This table concerns cases with $\Delta = \beta$. Of the cases considered, the highest load is possible with traffic uniformly divided over the superstations, with more time-critical than non time-critical traffic and with more high than low priority non time-critical traffic (row 3). The smallest stability bound is found in cases with relatively much low priority non time-critical traffic concentrated at one station (e.g., last row). Note that this table contains examples in which the asymptotic value of the stability bound as $L \to \infty$ is larger than the stability bound for some corresponding finite batch sizes (e.g., row 1 and row 7). The sufficient condition based on (5) and (15) (see the column with header "SC") is quite conservative again in some cases.

Table 17: Stability bound ρ^* for a twelve-station system with mixed traffic.

basic batch sizes												$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L\to\infty$	SC
1	1	1	1	1	1	1	1	1	1	1	1	.8571	.8623	.8727	.8727	.8727	.8633	.8219
4	2	1	4	2	1	4	2	1	4	2	1	.8871	.8912	.8912	.8889	.8882	.8861	.8284
7	4	1	7	4	1	7	4	1	7	4	1	.8930	.8972	.8944	.8937	.8947	.8922	.8304
7	1	1	7	1	1	7	1	1	7	1	1	.8834	.8906	.8871	.8898	.8878	.8889	.8295
4	1	1	4	1	1	4	1	1	4	1	1	.8889	.8903	.8880	.8884	.8872	.8856	.8276
1	4	1	1	4	1	1	4	1	1	4	1	.8868	.8843	.8834	.8814	.8788	.8759	.8276
1	1	4	1	1	4	1	1	4	1	1	4	.8300	.8290	.8348	.8348	.8348	.8305	.8108
4	1	1	1	4	1	4	1	1	1	4	1	.8869	.8794	.8834	.8804	.8791	.8759	.8276
4	1	1	1	1	4	4	1	1	1	1	4	.8421	.8446	.8421	.8409	.8401	.8382	.8108
1	4	1	1	1	4	1	4	1	1	1	4	.8571	.8546	.8491	.8458	.8436	.8392	.8108
7	1	1	1	1	1	1	1	1	1	1	1	.8372	.8372	.8372	.8372	.8372	.8372	.8257
1	7	1	1	1	1	1	1	1	1	1	1	.8700	.8710	.8632	.8654	.8654	.8531	.8257
1	1	7	1	1	1	1	1	1	1	1	1	.8182	.8182	.8151	.8182	.8182	.8072	.7826
1	1	7	1	1	1	1	1	7	1	1	1	.8205	.8166	.8166	.8188	.8191	.8136	.7947
1	1	13	1	1	1	1	1	1	1	1	1	.7912	.7843	.7869	.7890	.7895	.7818	.7643

7 Conclusions

This paper has demonstrated that the stability bound for communication systems with a timer-controlled token passing mechanism as Medium Access Control protocol, with multiple priority levels for traffic limited by the actual token rotation time, may depend in a non trivial way on the arrival processes and on the ratio of the ring latency and the transmission time. In case of random transmission times or token-passing times, the stability bound may also depend on the distributions of these quantities. An iterative procedure has been developed to obtain the large batch size limit for general systems with mixed traffic.

References

[1] ANSI/IEEE Standard 802.4-1985 "Token-Passing Bus Access Method and Physical Layer Specifications," The Institute of Electrical and Electronic Engineers, Inc., New York, 1985.

[2] "FDDI Token Ring Media Access Control", ANSI Standard X3.139, 1987.

[3] "FDDI Hybrid Ring Control", Draft Proposed American National Standard, ANSI X3T9.5, Rev. 6, May 11, 1990.

[4] E. Altman, Z. Liu. On the stability of timed token rings, *Performance Evaluation* **26** (1996), 219–234.

[5] J.P.C. Blanc, Performance evaluation of polling systems by means of the power-series algorithm, *Annals Oper. Res.* **35** (1992) 155–186.

[6] J.P.C. Blanc, Performance analysis and optimization with the power-series algorithm, in: *Performance Evaluation of Computer and Communication Systems*, eds. L. Donatiello, R. Nelson (Lecture Notes in Computer Science **729**, Springer, Berlin, 1993) 53–80.

[7] J.P.C. Blanc, L. Lenzini, Analysis of communication systems with timed-token protocols using the power-series algorithm, *Performance Evaluation* **27 & 28** (1996), 391–409.

[8] M. Conti, E. Gregori and L. Lenzini, Metropolitan Area Networks (MANs): protocols, modeling and performance evaluation, in: *Performance Evaluation of Computer and Communication Systems*, eds. L. Donatiello, R. Nelson (Lecture Notes in Computer Science **729**, Springer, Berlin, 1993) 81–120.

[9] J.G. Dai, S.P. Meyn. Stability and convergence of moments for multiclass queueing networks via fluid limit models, *IEEE Trans. Automat. Contr.* **AC-40** (1995), 1889–1904.

[10] C. Fricker, M.R. Jaïbi. Monotonicity and stability for periodic polling models, *Queueing Systems* **15** (1994), 211–238.

[11] P.J. Kühn, Multi-queue systems with non-exhaustive cyclic service, *Bell Syst. Techn. J.* **58** (1979), 671–698.

[12] I. Rubin and J.C.-H. Wu, Throughput and stability analysis of FDDI networks under nonsymmetric load, *Proc. IEEE GLOBECOM'93,* Houston, 1993, 1154–1158.

[13] M. Tangemann, Mean waiting time approximations for FDDI, in: *The Fundamental Role of Teletraffic in the Evolution of Telecommunications Networks*, eds. J. Labetoulle, J.W. Roberts (Elsevier, Amsterdam, 1994) 973–984.